UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical & Computer Engineering

ECE 204 *Numerical methods*

Approximating the integral using least-squares best-fitting polynomials

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Introduction

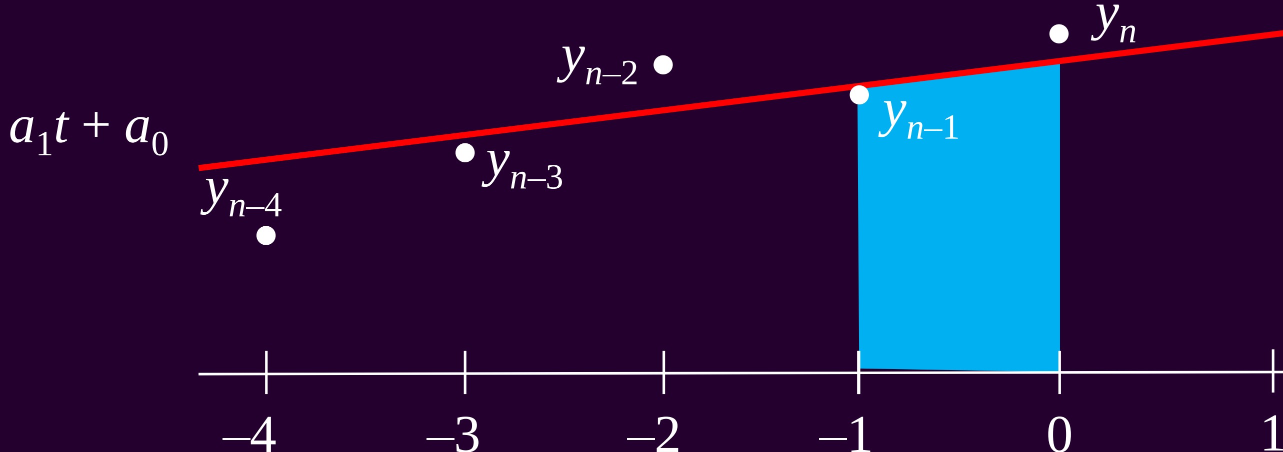
- In this topic, we will
  - Discuss how to estimate an integral of data by using the least-squares best-fitting polynomials
  - Estimating $\int_{t_{n-1}}^{t_n} y(t)\,\mathrm{d}t$ or $\int_{t_n}^{t_{n+1}} y(t)\,\mathrm{d}t$ where $t_k = t_0 + kh$
  - Describe the formula for both linear and quadratic polynomials

Approximating the integral

- Suppose we have found the least-squares linear polynomial that passes through N equally-spaced points
  - We want to integrate that line over the last time interval
  - The least-squares linear polynomial is $a_1 t + a_0$ so we integrate

$$\int_{-1}^{0} \left(a_1 t + a_0 \right) \mathrm{d}t = a_0 - \frac{a_1}{2}$$

  - Again, we scaled, and thus we must account for this: $\int_{t_n}^{t_{n+1}} y(t)\,\mathrm{d}t \approx h\left(a_0 - \frac{a_1}{2} h \right)$

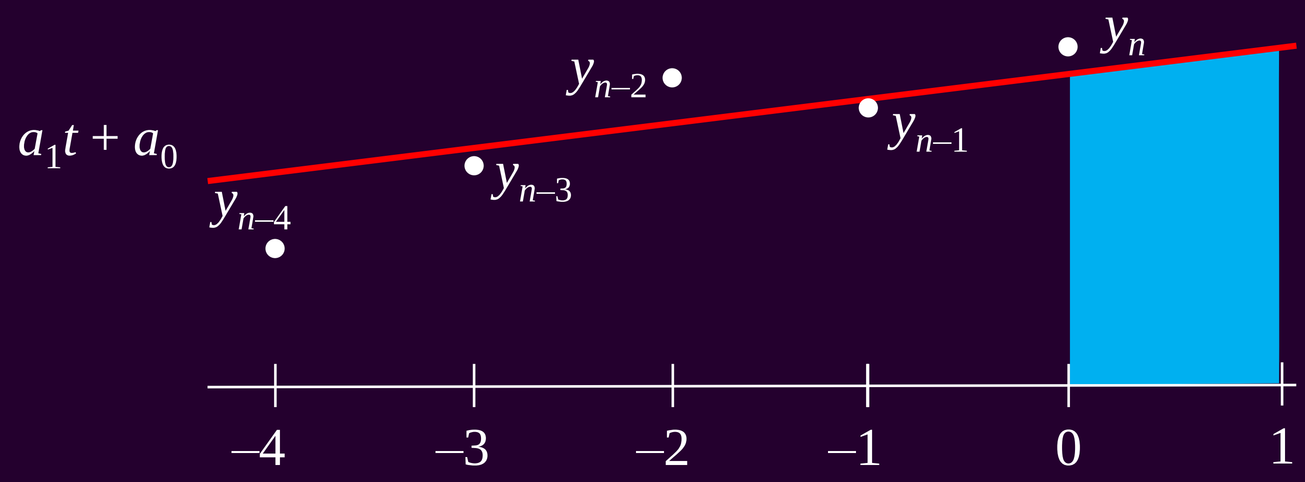


Approximating the integral

- Similarly, we can estimate the integral over the next time interval:

$$\int_0^1 (a_1 t + a_0)\,\mathrm{d}t = a_0 + \frac{a_1}{2}$$

  – Once again, we account for scaling: $\int_{t_n}^{t_{n+1}} y(t)\,\mathrm{d}t \approx \left(a_0 + \frac{a_1}{2}\right)h$

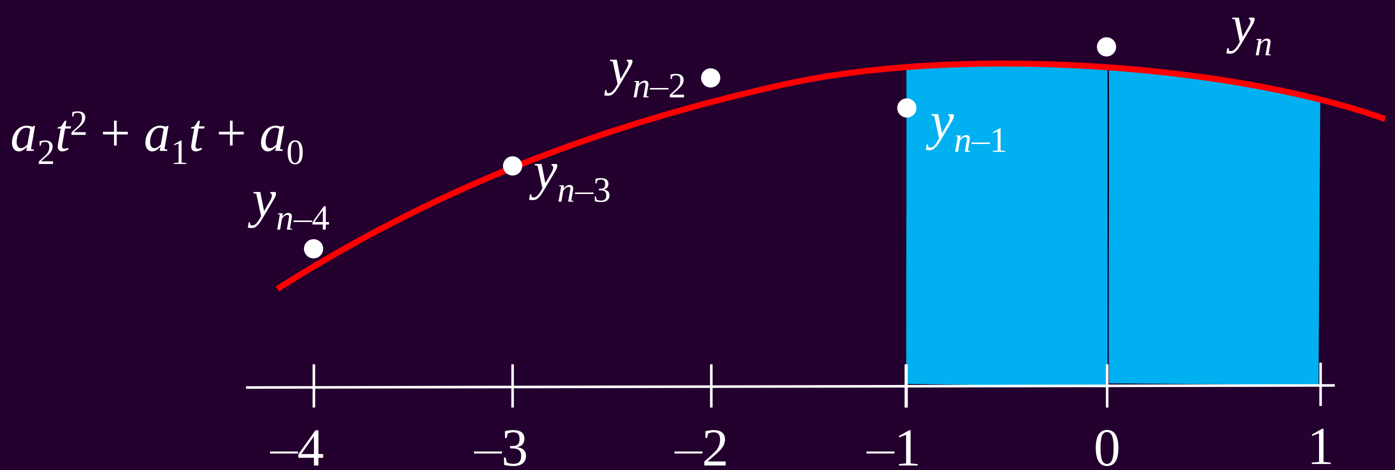


Approximating the integral

- We can perform the same operation for a least-squares quadratic polynomial

$$\int_{t_{n-1}}^{t_n} y(t)\,\mathrm{d}t \approx h\int_{-1}^{0}\left(a_2t^2 + a_1t + a_0\right)\mathrm{d}t = h\left(a_0 - \frac{a_1}{2} + \frac{a_2}{3}\right)$$

$$\int_{t_n}^{t_{n+1}} y(t)\,\mathrm{d}t \approx h\int_{0}^{1}\left(a_2t^2 + a_1t + a_0\right)\mathrm{d}t = h\left(a_0 + \frac{a_1}{2} + \frac{a_2}{3}\right)$$



Summary

- Following this topic, you now
  - Understand how to estimate the integral using least-squares best-fitting polynomials
  - Are aware that we can both estimate the integral over the last time interval, or extrapolate and estimate the integral over the next time interval
  - Understand that if we already have the coefficients, we can find these estimates in O(1) time

References

[1] https://en.wikipedia.org/wiki/Least_squares

Acknowledgments

None so far.

Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`. Mathematical equations are prepared in MathType by Design Science, Inc.

Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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